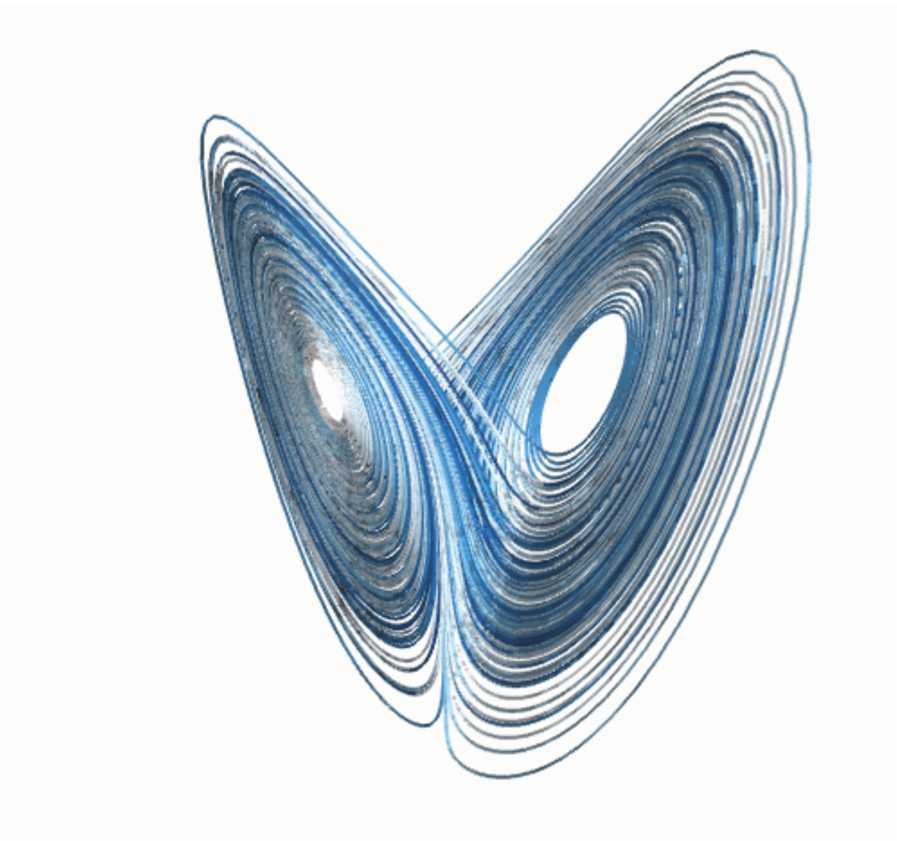
Predictability of wall-bounded flows by massive ensemble forecasting

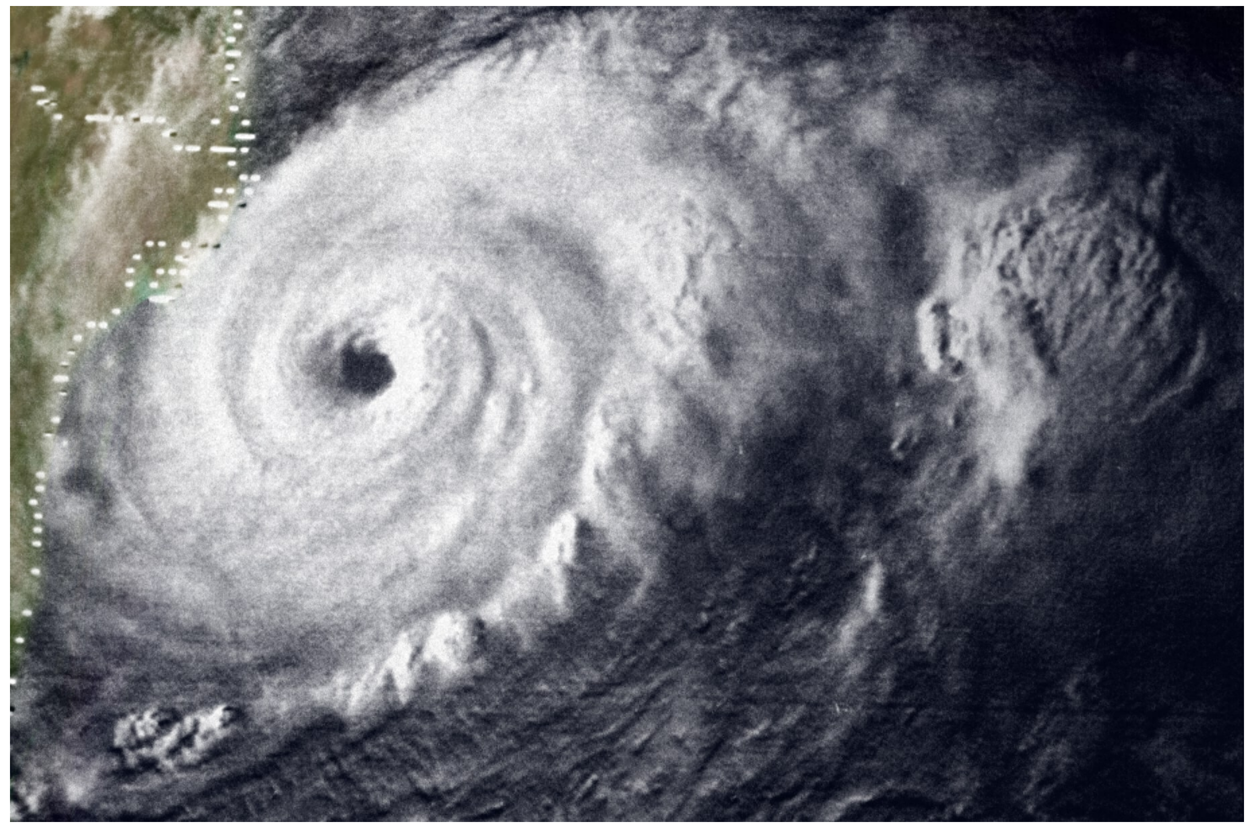
Alberto Vela-Martín[1] & Miguel P. Encinar[1]

Universidad Carlos III de Madrid

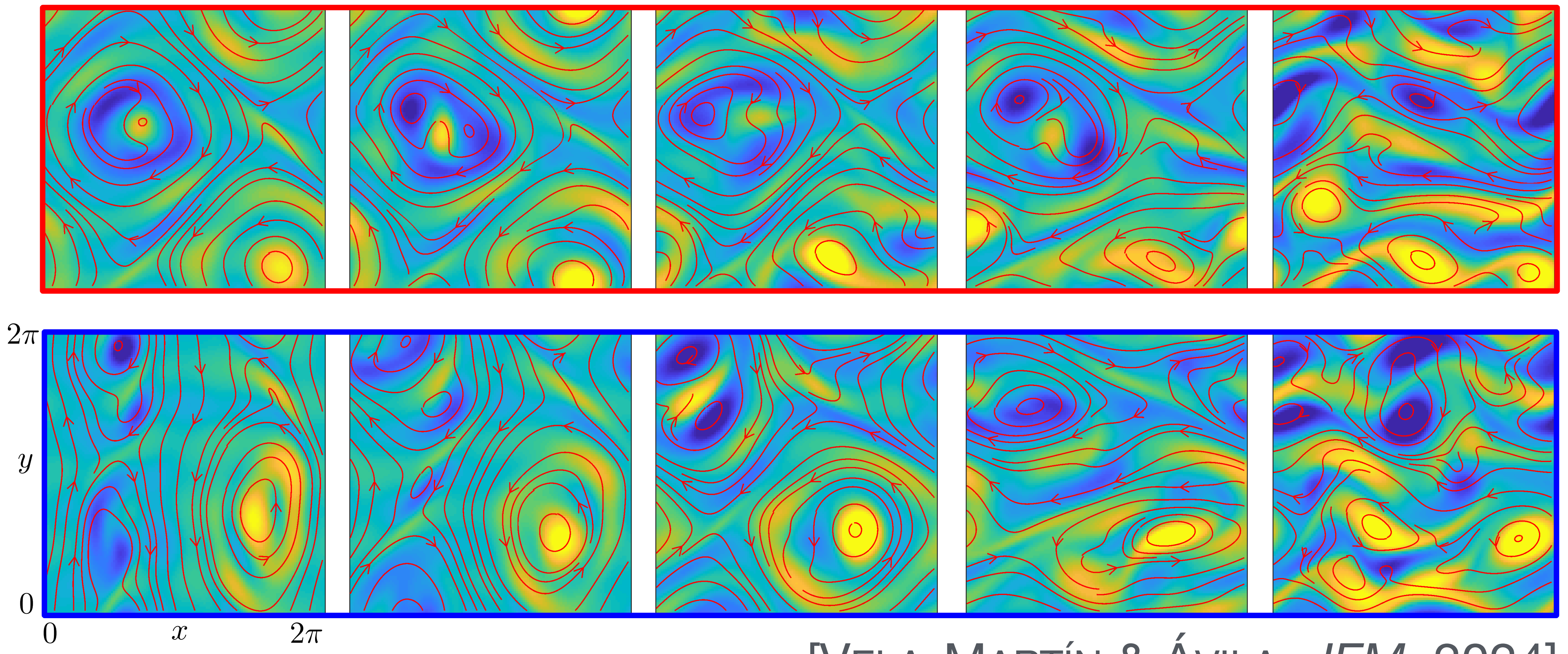
Predictability of Turbulent Flows

Chaos

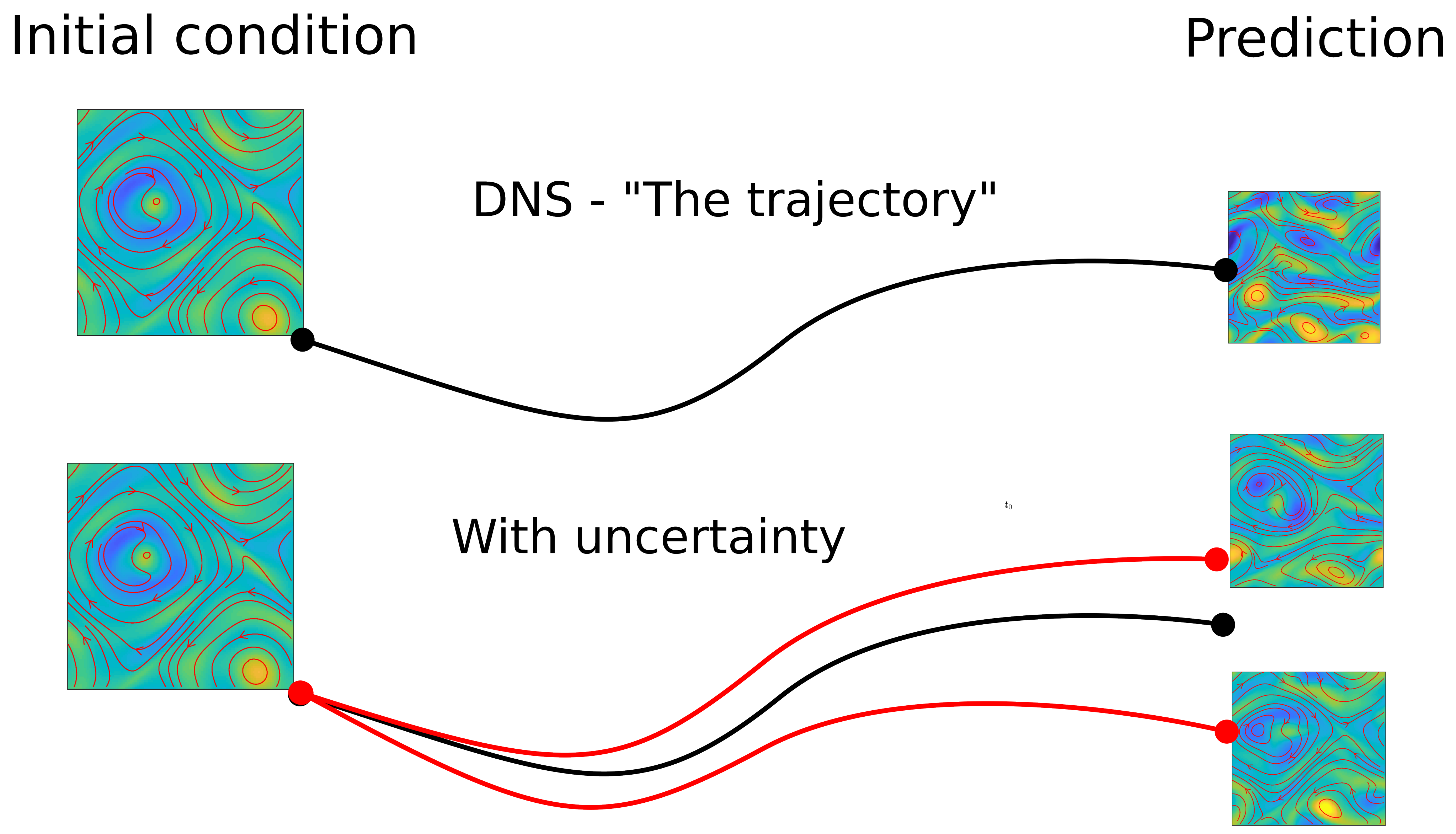




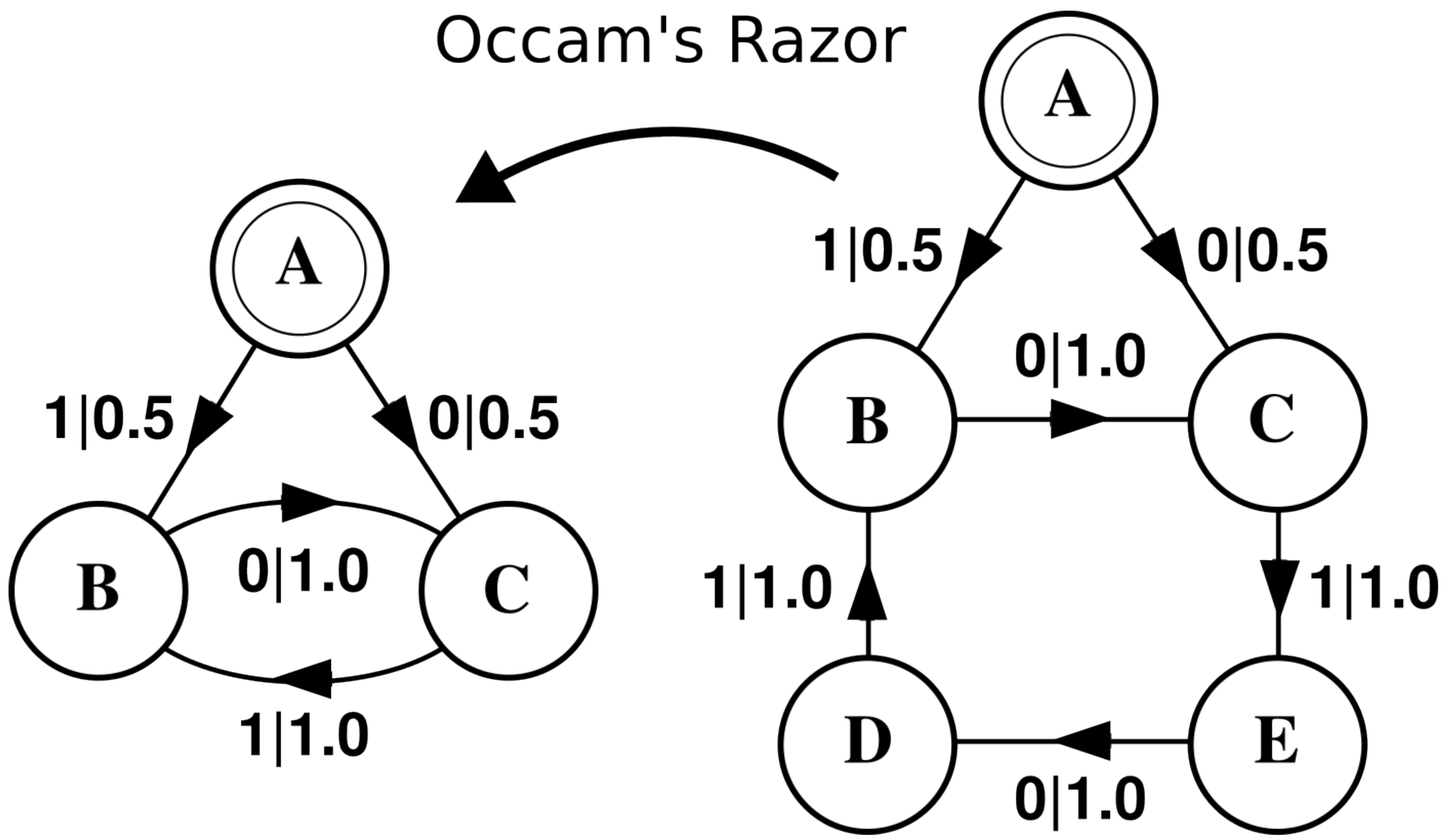
Extreme Events



[VELA-MARTÍN & ÁVILA, *JFM*, 2024]

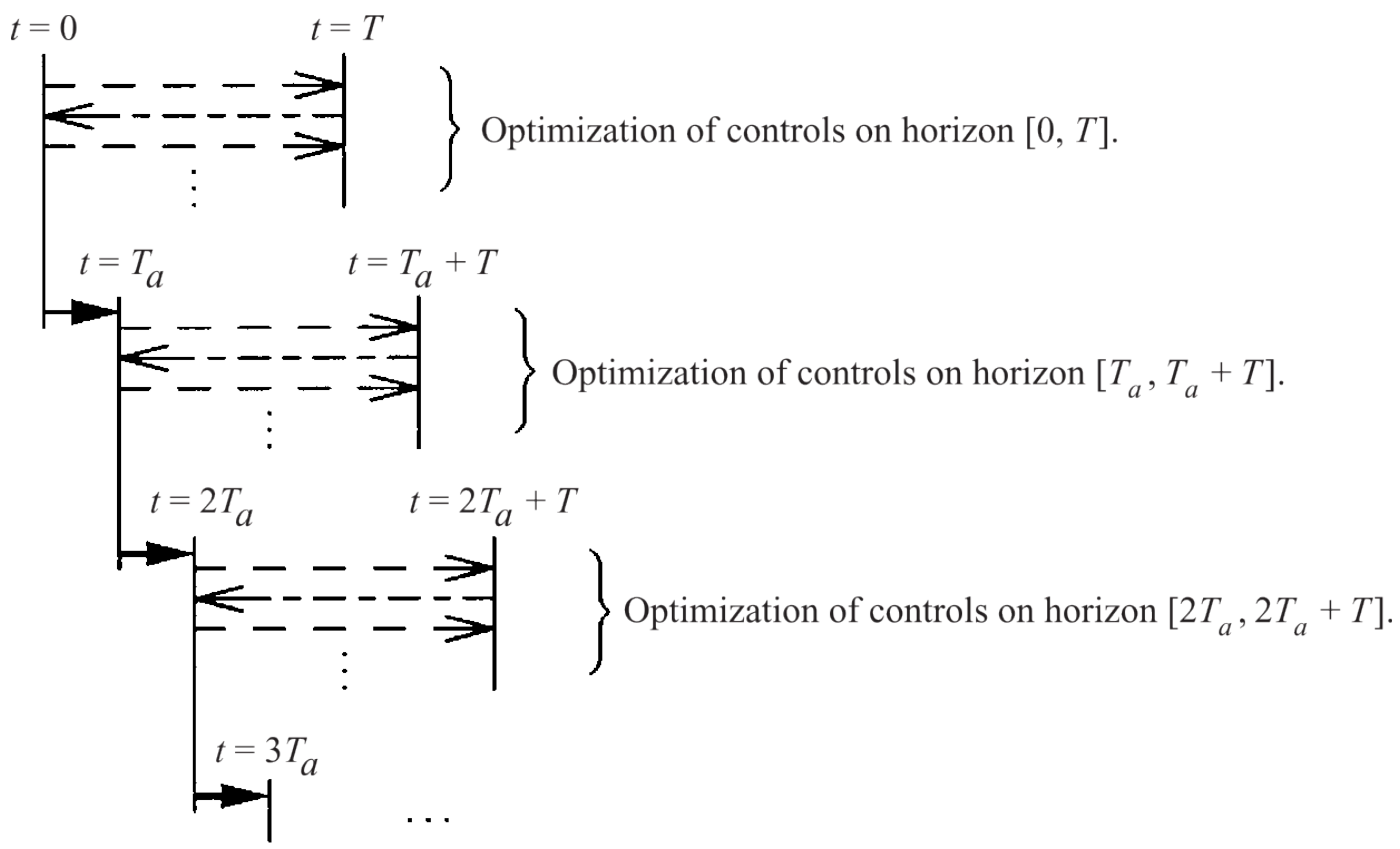


Optimal Predictive Models



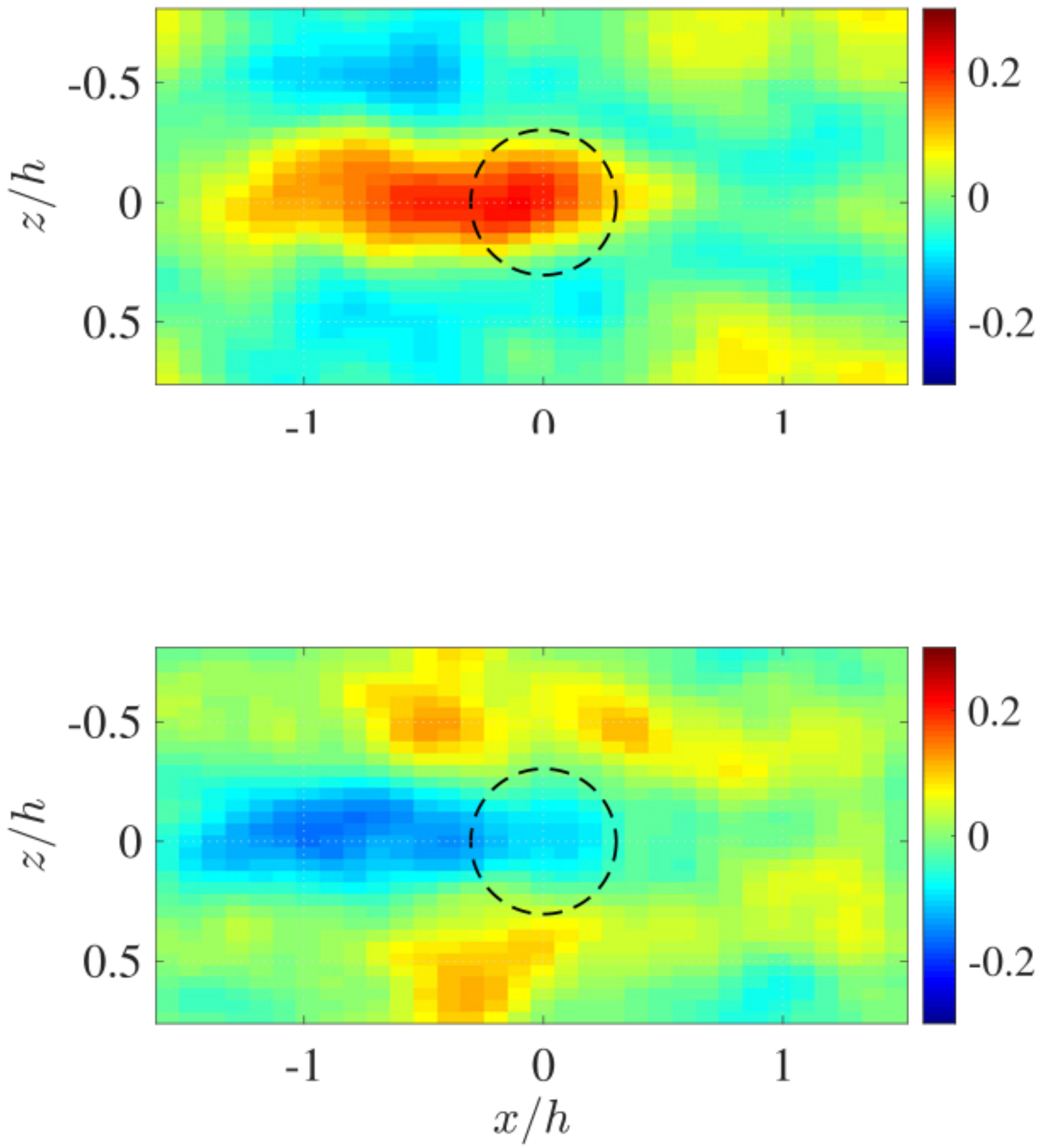
Implications for Control

Optimal Predictive Control



[BEWLEY ET AL, *JFM*, 2001]

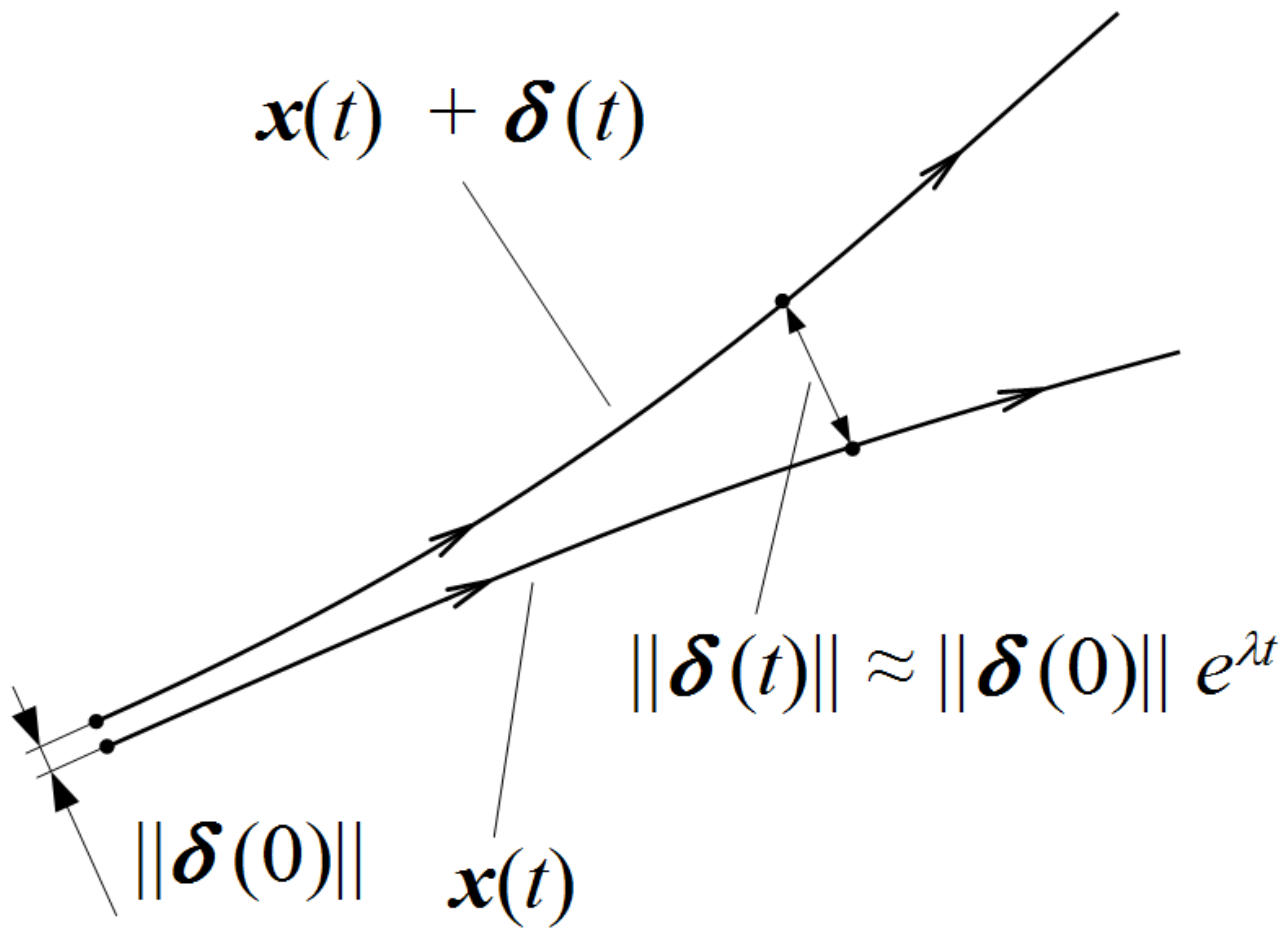
Monte-Carlo Control



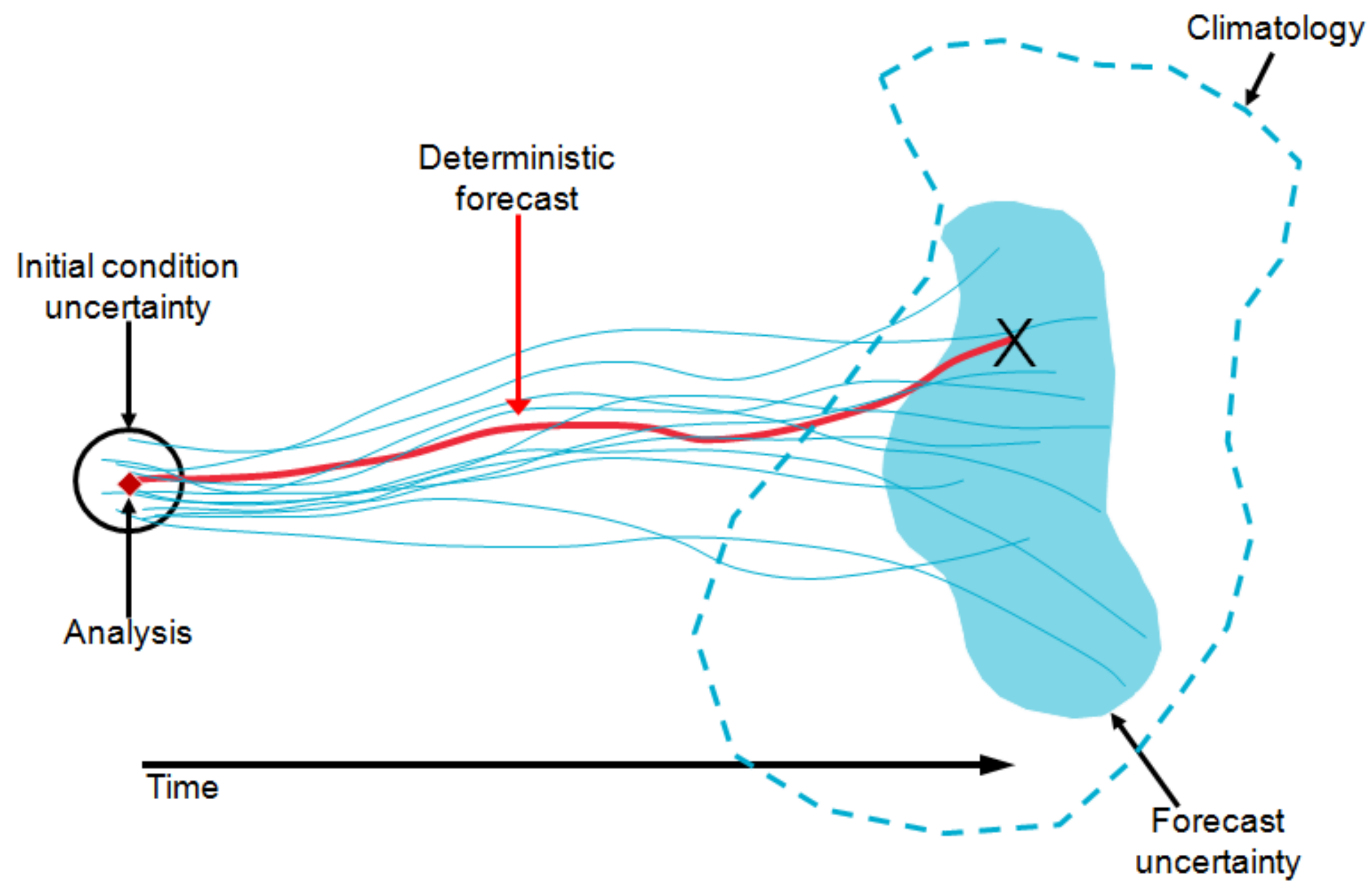
[PASTOR ET AL., *JPCS*, 2020]

How to measure predictability and uncertainty growth

Lyapunov Exponents

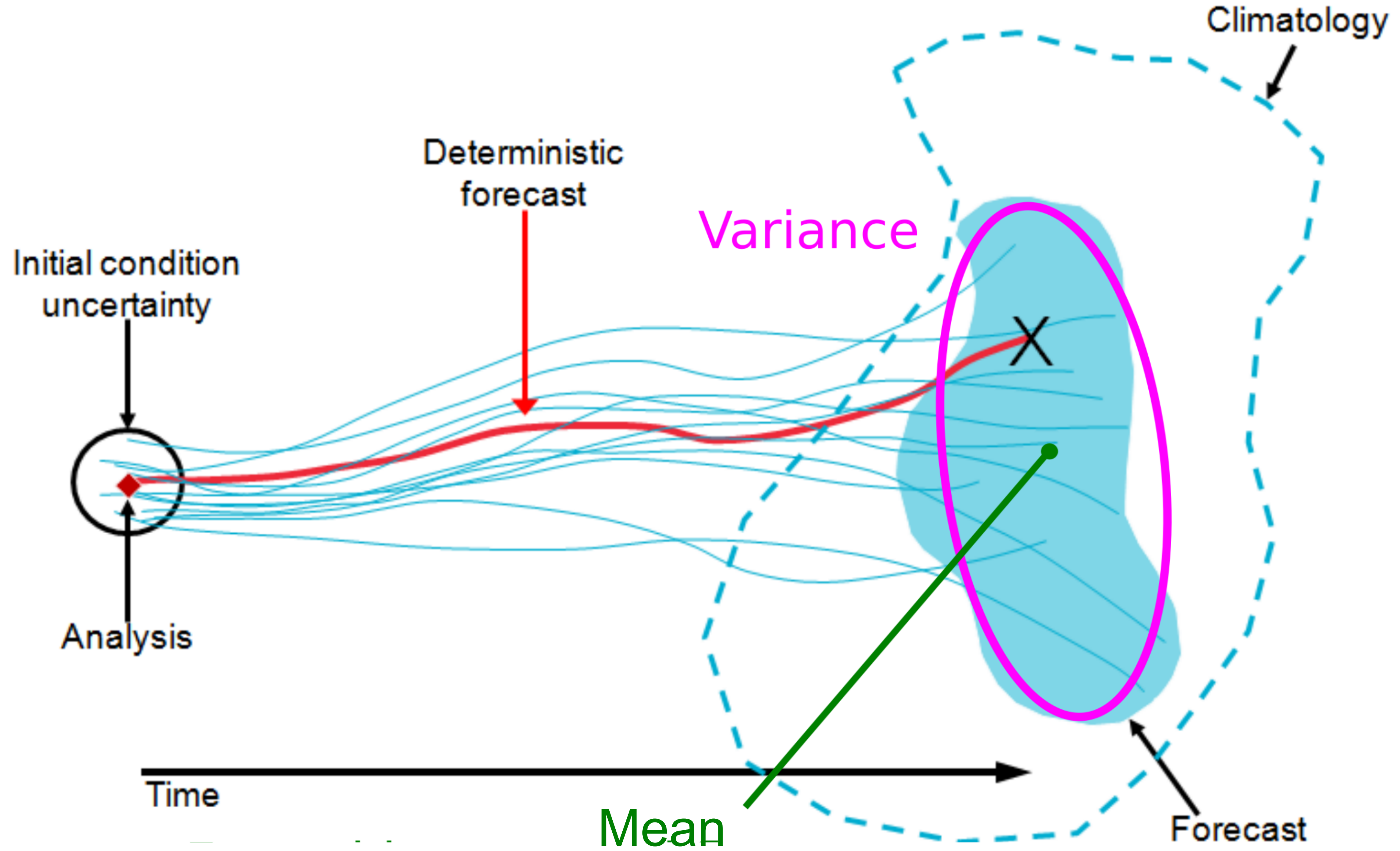


Ensemble Forecasting

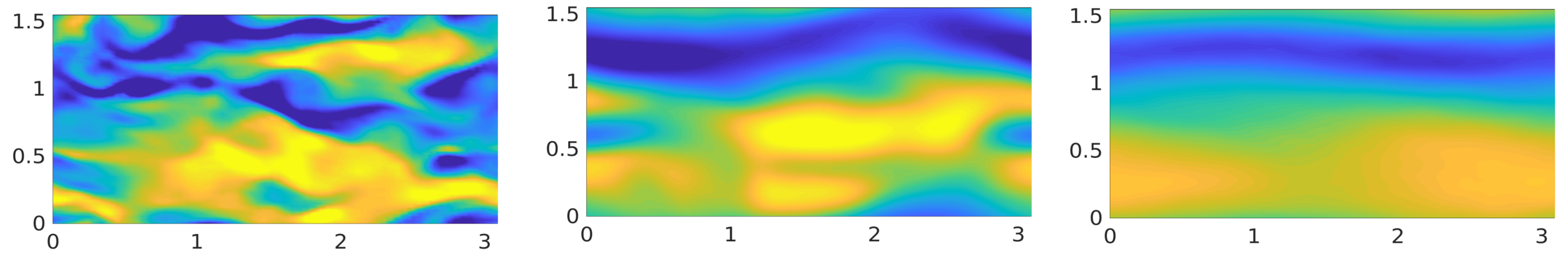


Massive ensemble forecasting of turbulent channel flow

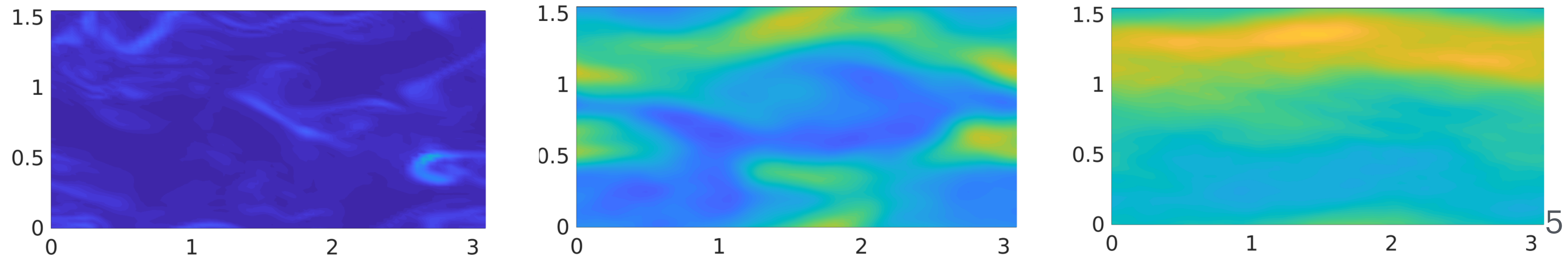
- **‘Lyapunov’ ensembles**—Small initial perturbation
- Ensembles with 1024 **DNS**s (trajectories)
- $\boldsymbol{Re_\tau = 180}$, small box $\boldsymbol{(\pi \times \pi/2)}$
- **96** initial base flows (100 000 simulations)
- Ensemble-averaged flow field—**Optimal prediction** in the least squared sense
- Unpredictable flow patterns are **filtered out**



Ensemble averaged streamwise velocity fluctuations



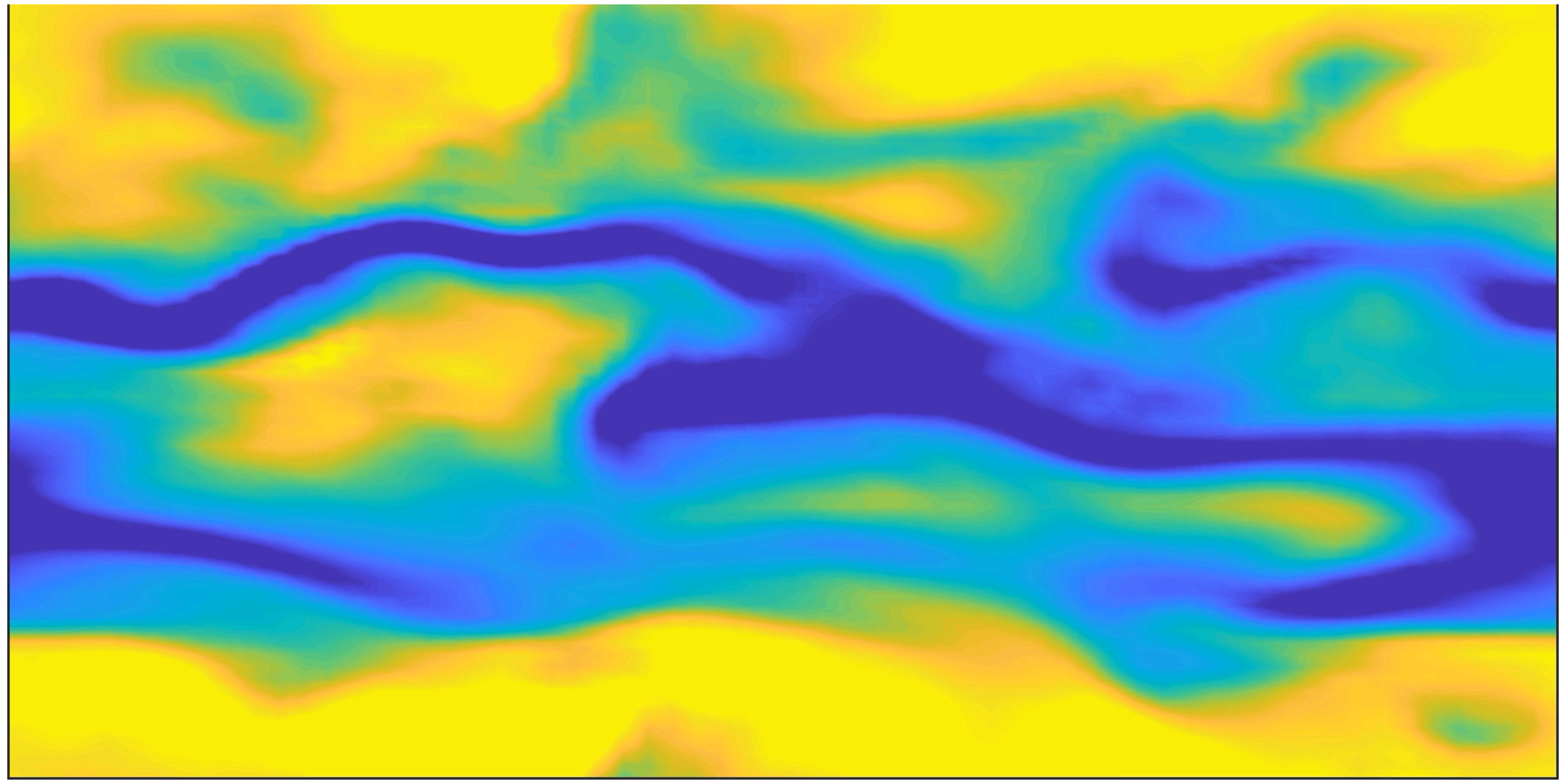
Ensemble variance streamwise velocity fluctuations



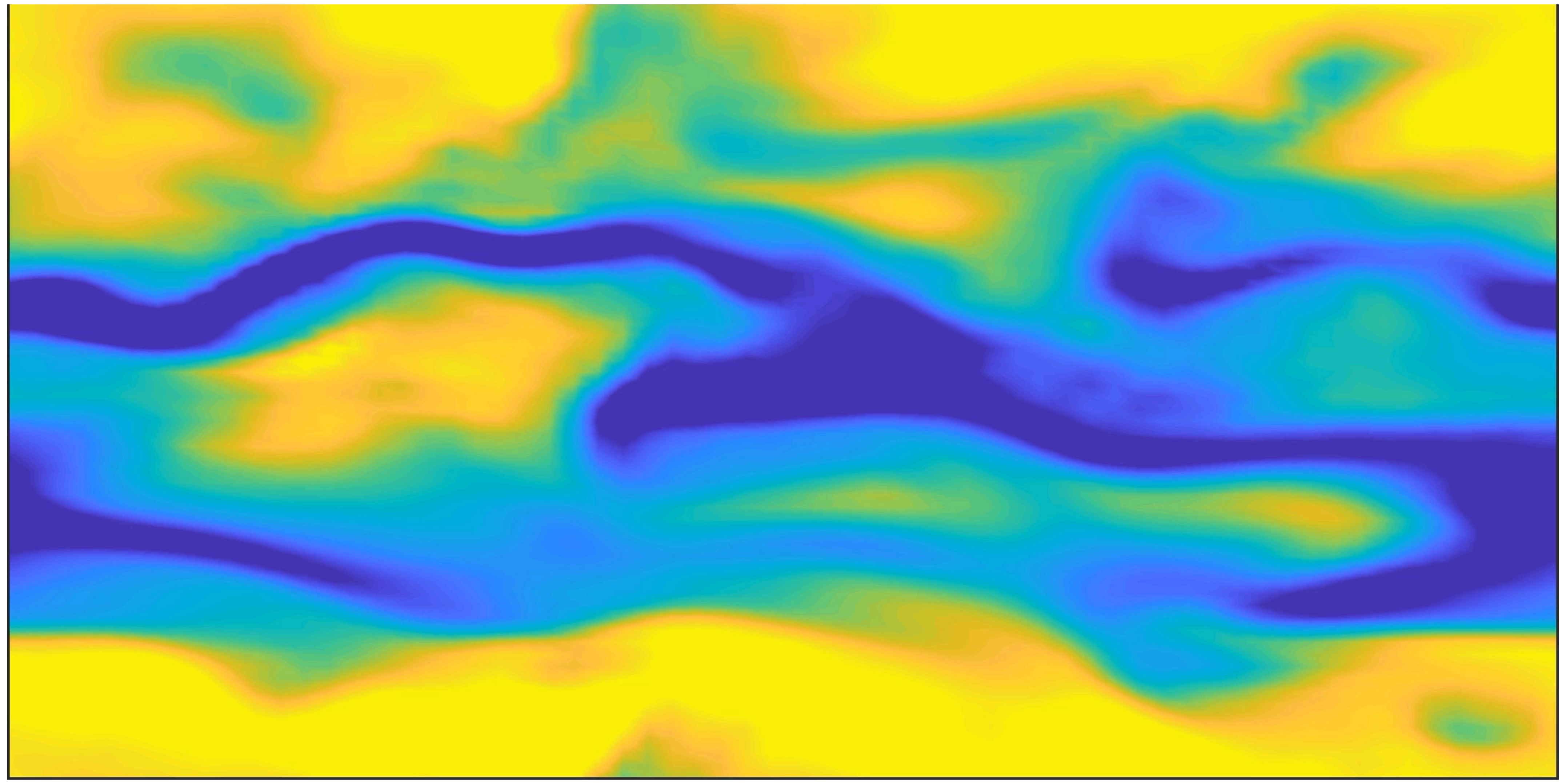
How does it look like?

Video of u of simulation *#1* out of *#96* at $y^+ \approx 50$

Base Trajectory

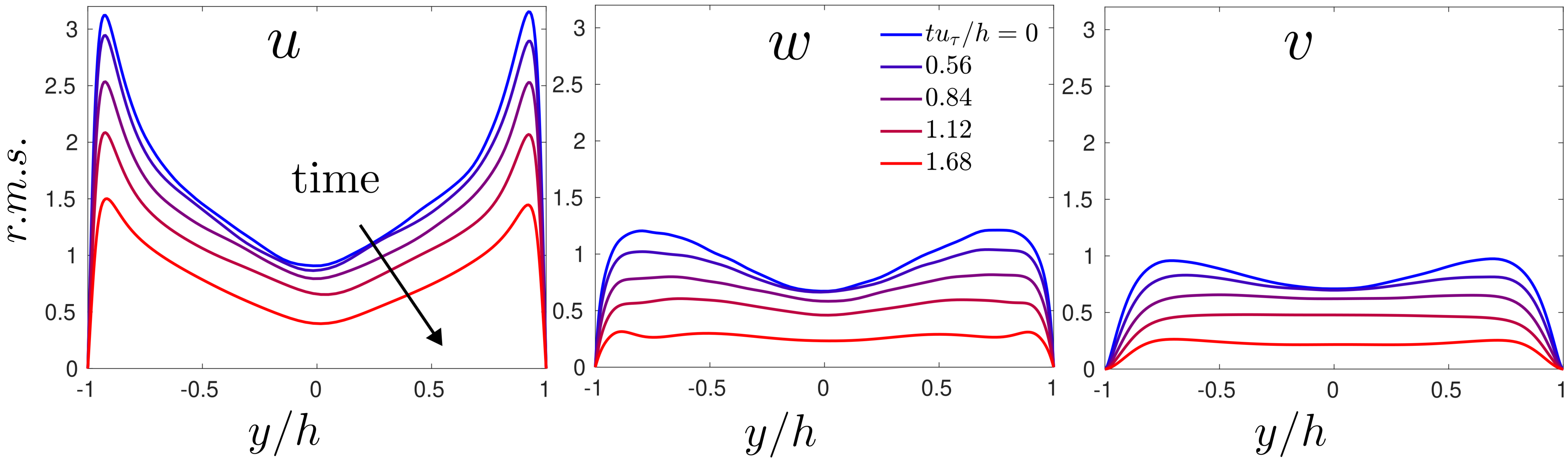


Ensemble Forecast



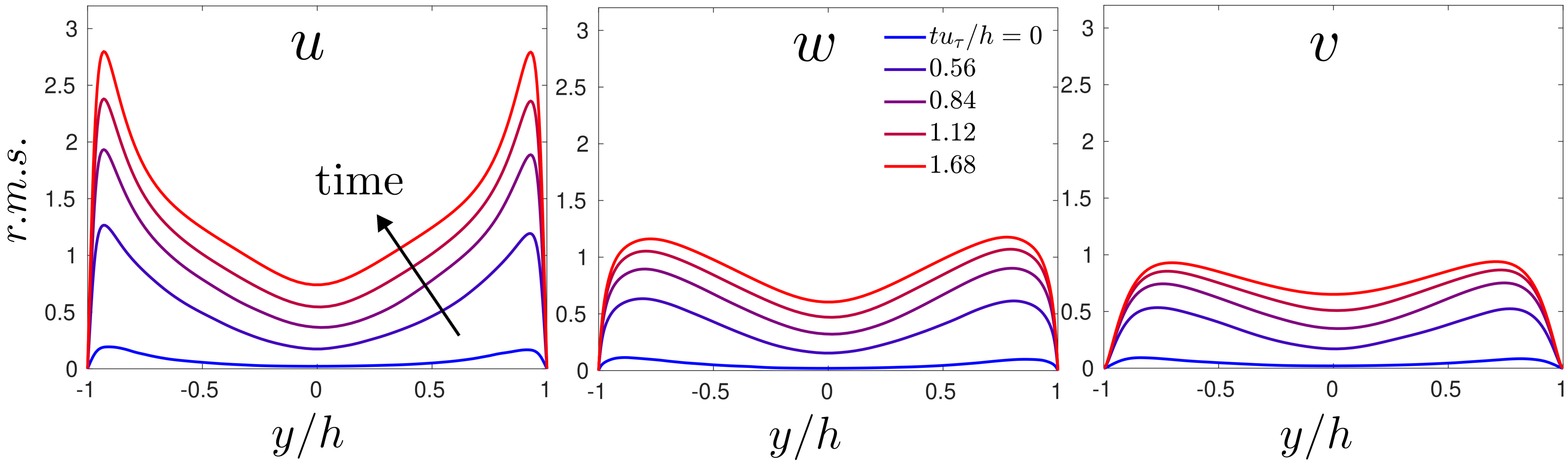
Predictable energy

r.m.s. of the ensemble-averaged velocity fluctuations



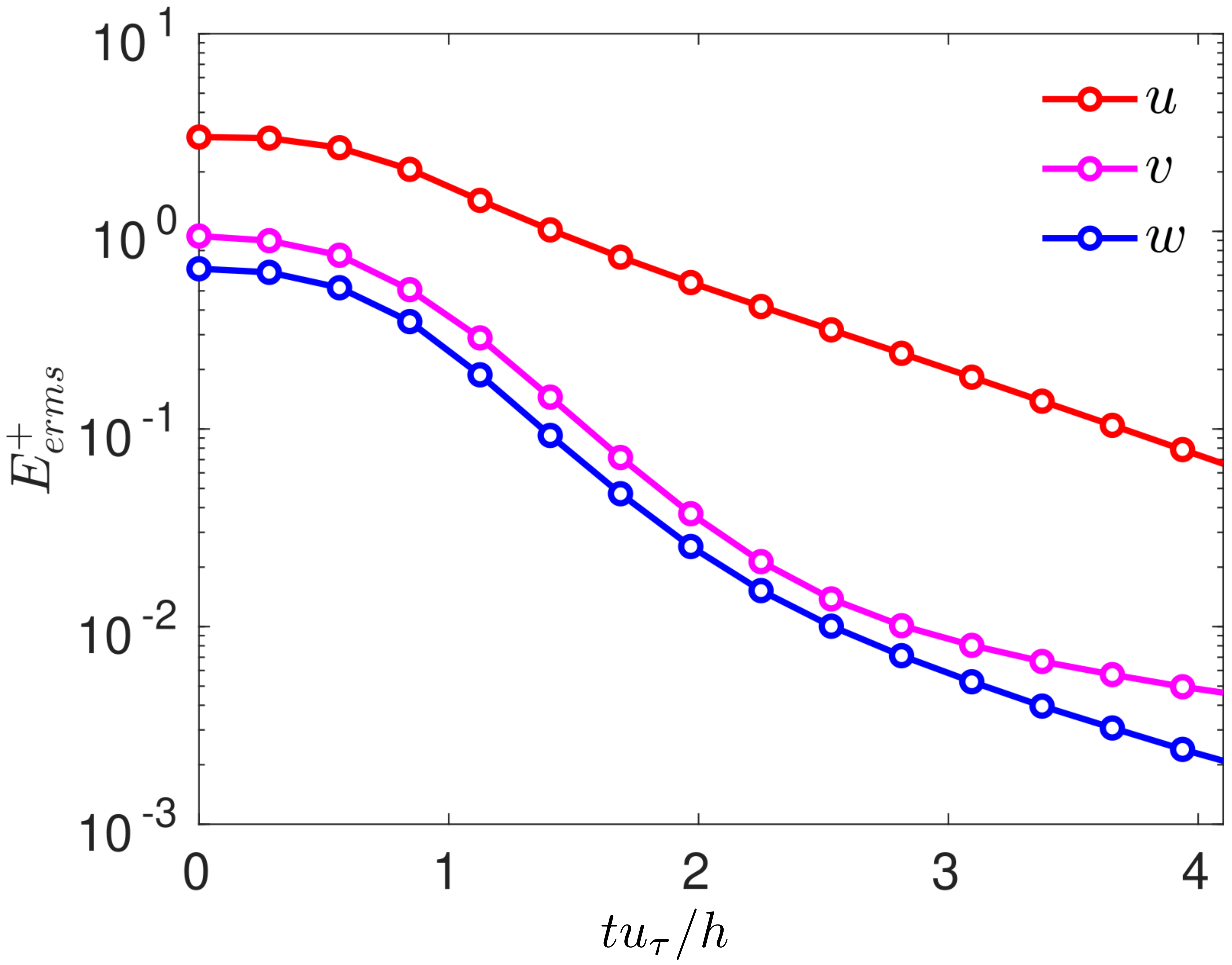
Uncertainty growth (Unpredictable energy)

Ensemble standard deviation of the velocity fluctuations

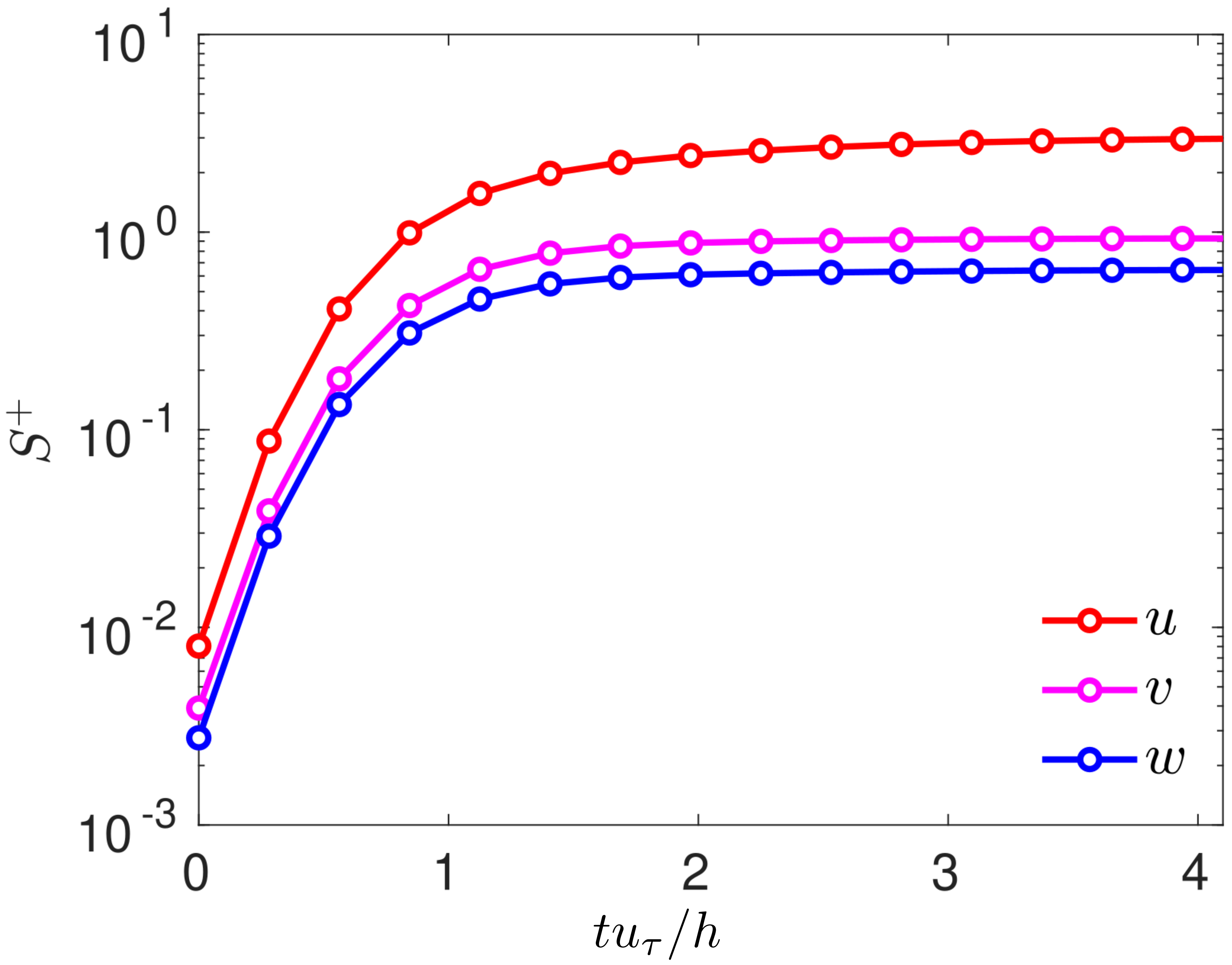


Predictable energy vs uncertainty

Energy of the ensemble averaged fluctuations



Ensemble-**variance** of the velocity fluctuations

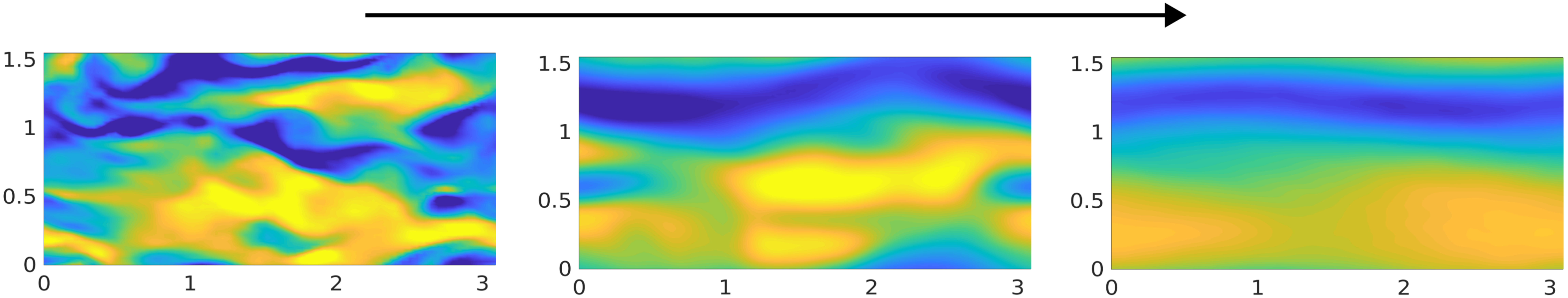


Modelling uncertainty growth

TARGET: Affordable predictability/uncertaity calculations

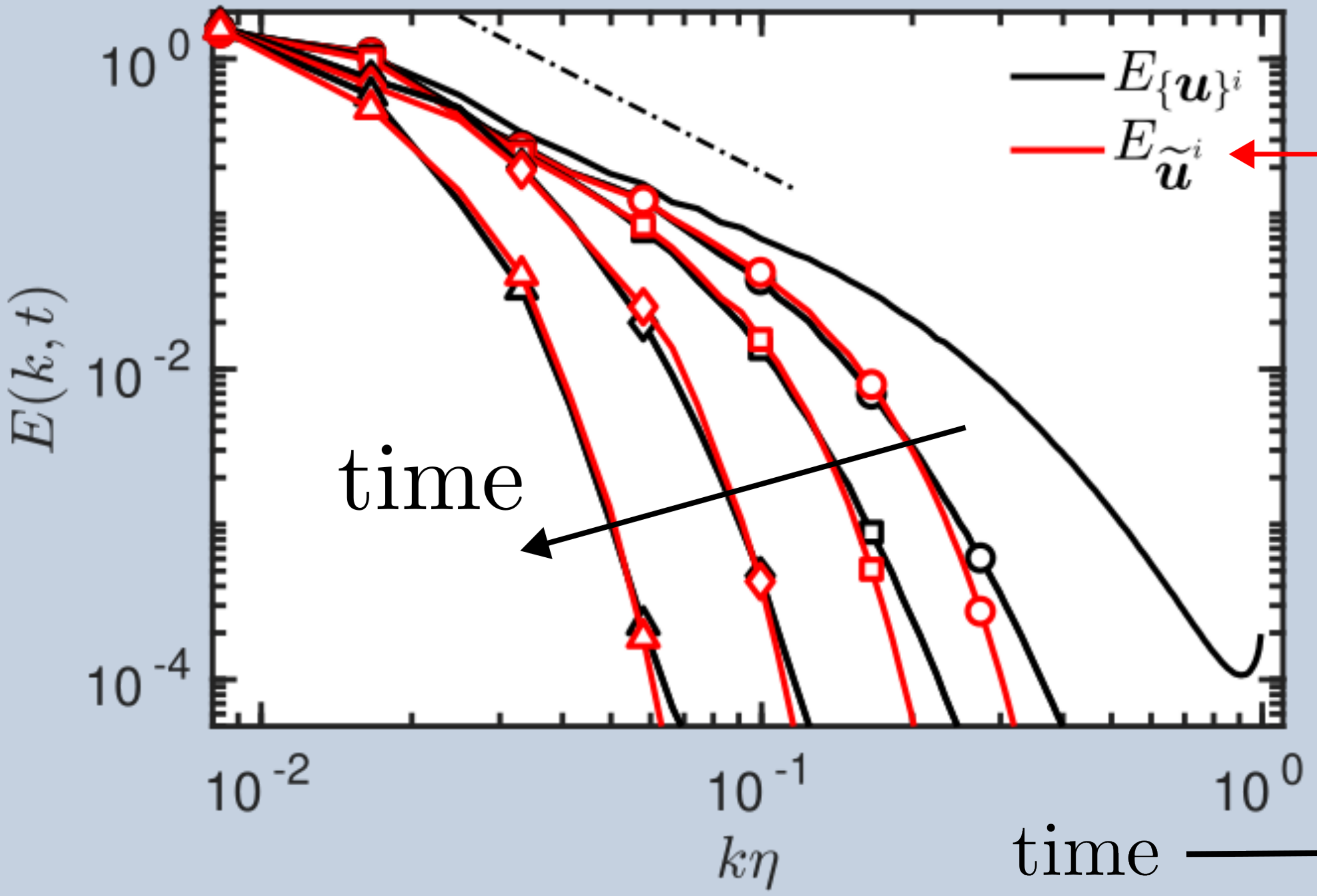
- Study the scaling of perturbation growth
- Is there a relatively simple model for uncertainty growth?

$$\text{Uncertainty} \approx f(\text{Time})$$



A simpler case: HIT

Uncertainty growth in isotropic turbulence



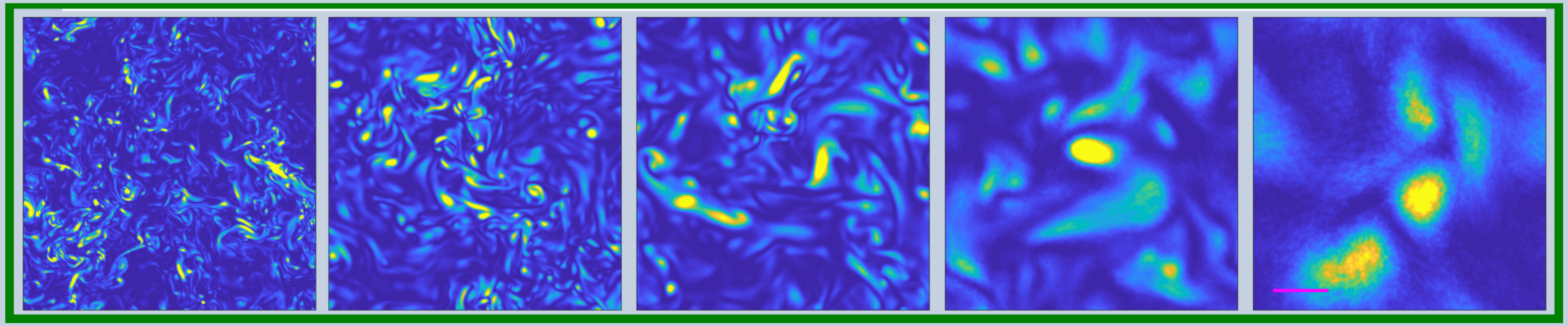
Predictability loss as a time-dependent diffusion process

$$\widehat{\boldsymbol{u}}_{\text{predict}} = \widehat{\boldsymbol{u}}_{\text{true}} \exp -\ell^2(t)k^2$$

Minimum predictable scale

$$\ell \sim \varepsilon^{1/2} t^{3/2}$$

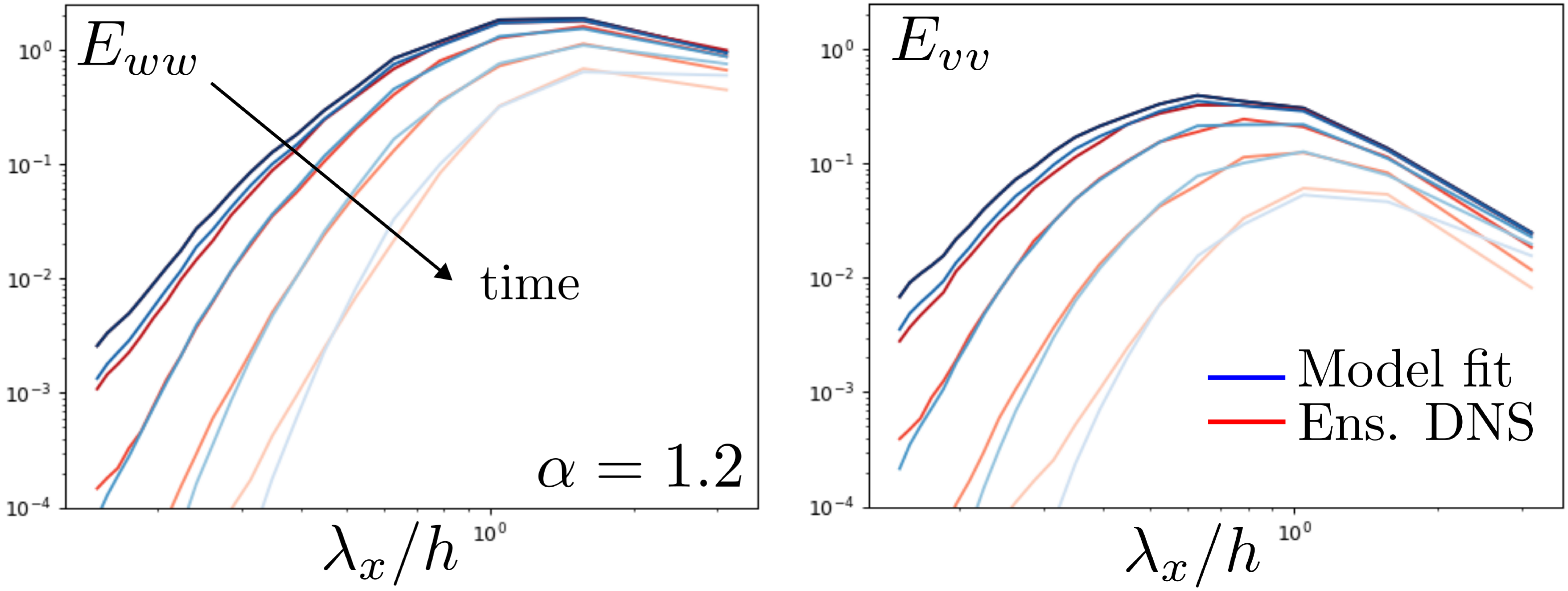
time



Predictability loss in channel flow

Cross-plane velocity components — Close to the wall

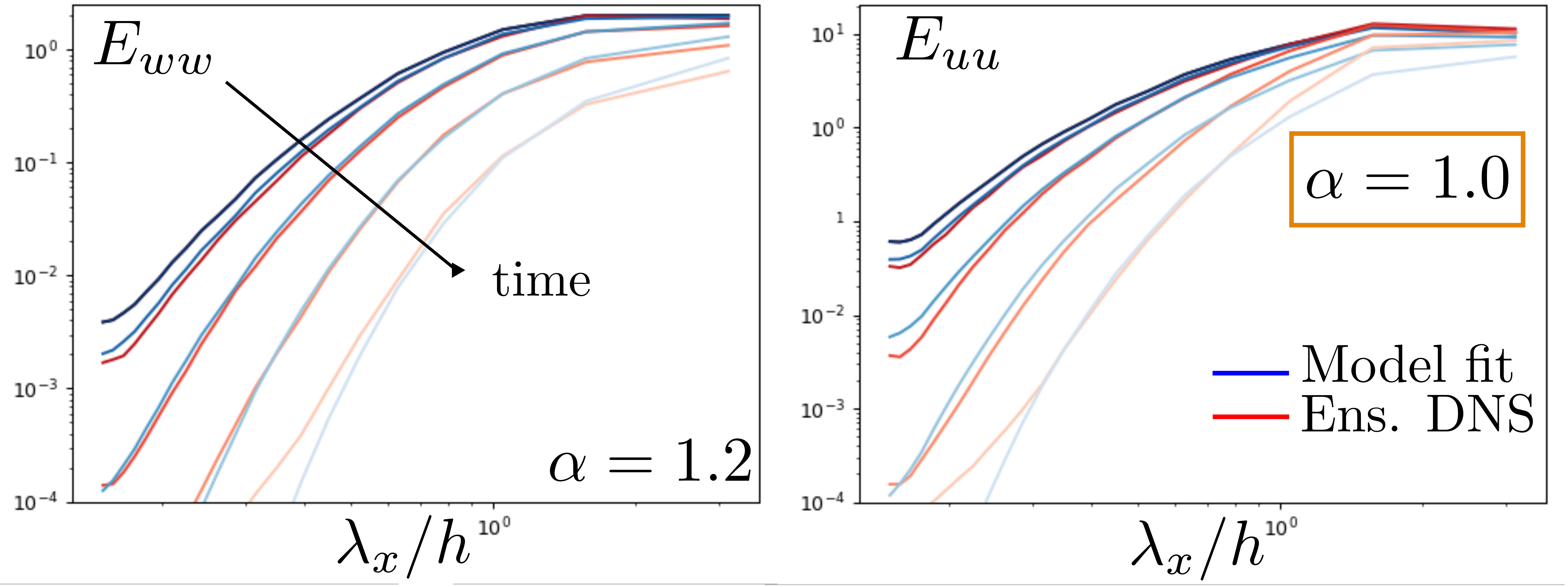
$y^+ = 10$ $\qquad \widehat{\boldsymbol{u}}_{\text{predict}} = \widehat{\boldsymbol{u}}_{\text{true}} \exp -\ell^\alpha |k_x|^\alpha$



Predictability loss in channel flow

The **streamwise** velocity component is **different**

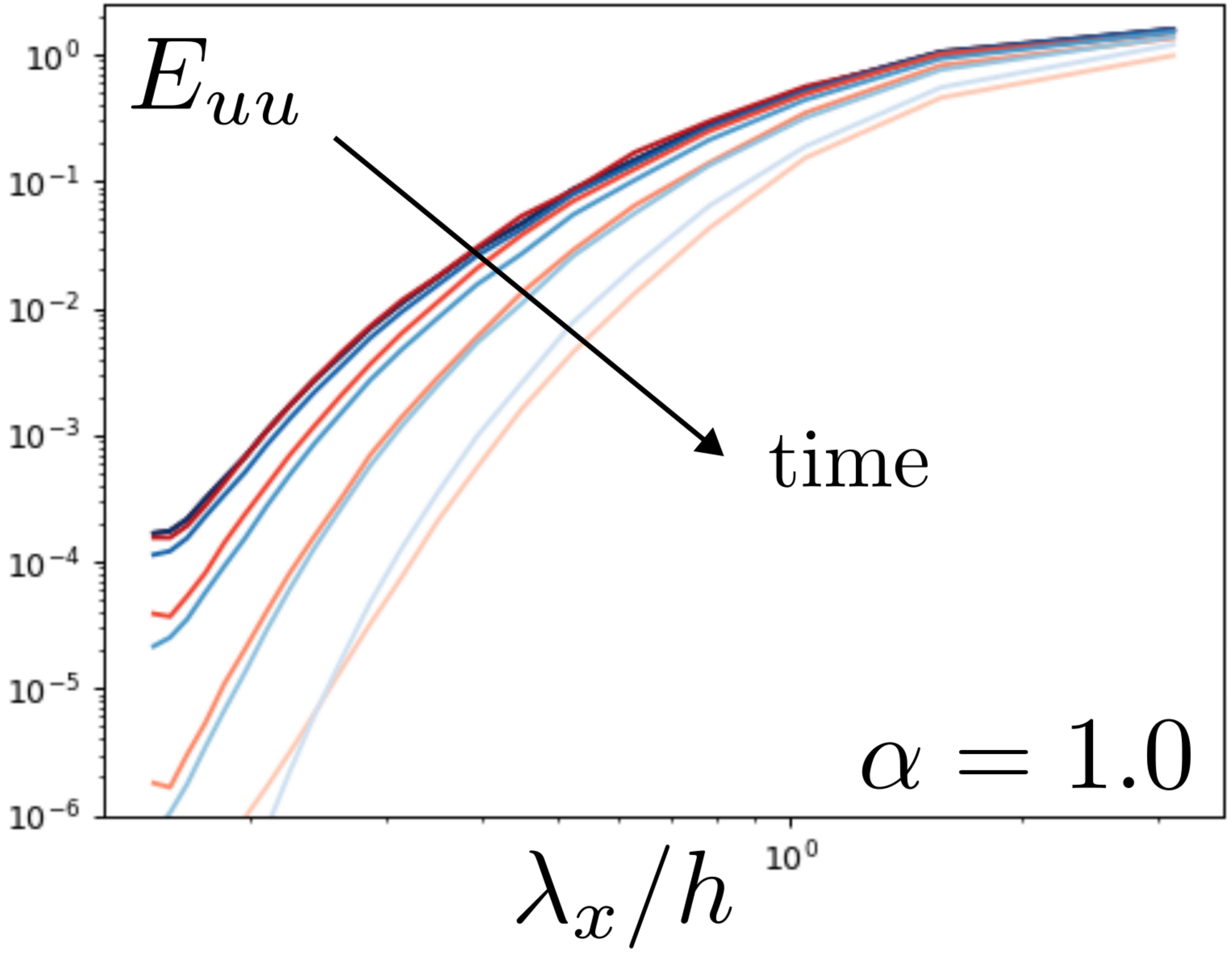
$y^+ = 32$ $\qquad \widehat{\boldsymbol{u}}_{\text{predict}} = \widehat{\boldsymbol{u}}_{\text{true}} \exp -\ell^\alpha |k_x|^\alpha$



Predictability loss in channel flow

The anomaly exponents α are consistent towards the channel centreline

$$y = h \quad \widehat{\boldsymbol{u}}_{\text{predict}} = \widehat{\boldsymbol{u}}_{\text{true}} \exp -\ell^{\alpha} |k_x|^{\alpha}$$



Conclusions and Future Work

- **Predictability loss** in wall-bounded flows seems to be well described by **anomalous diffusion** ($\alpha \neq 2$) with a **time-dependent diffusion coefficient**

- This diffusion is **different** for the streamwise and the cross-plane velocities. The source of the anomaly must be the **shear**…

- Similarities with **LES modelling,**

$$\partial_t\{u_i\} + \{u_j\}\partial_j\{u_i\} = -\partial_i\{p\} + \boxed{\partial_j \tau_{ij}} + \{f_i\},$$
$$\partial_i\{u_i\} = 0,$$

$$\tau_{ij} = \{u_i\}\{u_j\} - \{u_i u_j\}$$

- **Potential**
  - DNS-informed control with uncertainty
  - LES models from optimal prediction over the uncertain (subgrid) scales

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Thanks for listening!

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Questions?

- **Potential**
  - DNS-informed control with uncertainty
  - LES models from optimal prediction over the uncertain (subgrid) scales